June 2023

I am a mathematician…

I am a mathematician. I am curious and wish to solve problems but I know the answer is only the beginning. I recognise that getting stuck is part of problem-solving and that it helps me develop the resilience and strategies to persevere.
In order to become fluent in recognising, representing and communicating about mathematical concepts, I actively explore and analyse them through talk with others; making conjectures, justifying my ideas and generalising from specific examples to create an increasingly efficient and connected understanding. I apply this understanding and mathematical habits of mind to find patterns that help me break into, make sense of and break down novel problems to find solutions.
I reflect on what I have done to improve my strategies, collaborating with others to construct a shared understanding that increasingly helps us to make sense of the world, giving us enjoyment, agency and a sense of self and place.

The rights of the mathematician

1. The right to enjoy mathematics
2. The right to have interests and preferences
3. The right to make jottings, drawings and working out
4. The right to use our own methods and approaches
5. The right to use manipulatives and resources
6. The right to reason, to talk about maths and be listened **to**
7. The right to make mistakes
8. The right to estimate, to guess and to conjecture
9. The right to ponder and take time
10. The right to be playful

Gripton, C. (2020)

"Mathematics could be like roller-skating, but usually it's like being told to stop roller-skating and come in and tidy your room."
Richard Winter (1991)

"Teachers do not create learning, learners create learning, teachers create the conditions in which students learn."

Dylan Wiliam (2006)

Contents

[CTRL + Click to follow links]

Three core components of teaching to develop mathematicians

Appendix B – Curriculum overviews is a separate Powerpoint document which can be found in the same folder.

What is mathematics?

Mathematics is the science that deals with the logic of shape, quantity and arrangement. It can be seen as the search for meaningful pattern and structure that can be applied to model and solve problems in every facet of human enquiry. Its substantive knowledge has evolved from simple counting, measurement and calculation into increasingly abstract ideas from algebra to calculus and beyond into analysis and topology – content that is mostly beyond the scope of primary school learning. However, the disciplinary knowledge (including skills of inquiry, conjecture and proof) is an integral part of the study and application of mathematics in the early years and throughout the primary years.

Curriculum expectations

In the English national curriculum, outcomes in mathematics go beyond recalling facts and emulating procedures, as emphasised by the three aims of the curriculum and in the expectation that:

> *"By the end of each key stage, pupils are expected to* ***know****,* ***apply*** *and* ***understand*** *the matters, skills and processes specified in the relevant programme of study."*
>
> (DfE, 2014)

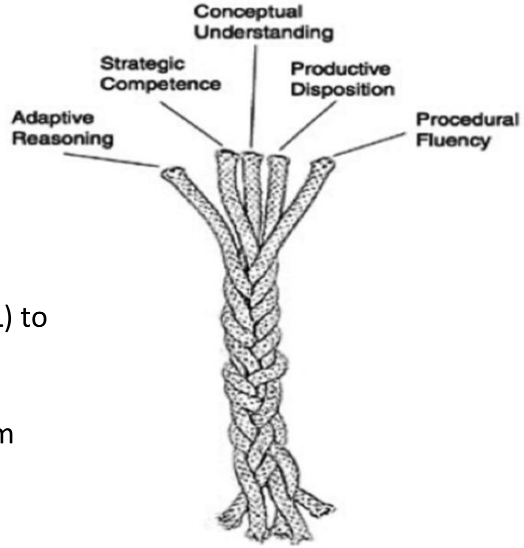
Our curriculum intends to make explicit the content knowledge and the necessary mathematical processes and social-emotional learning skills involved in learning mathematics so that teachers and leaders help all learners achieve the three aims of the national curriculum and to demonstrate all elements of "*mathematical proficiency"* (National Research Council, 2001) – *see image, right*.

Learners develop mathematical processes through working with the content of the curriculum. As they engage in applying these processes, together with social-emotional learning skills, they develop their understanding of the content. The processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- connecting
- communicating
- representing
- selecting tools and strategies

They are the means through which all learners develop and apply mathematical knowledge, concepts, and skills. In the Cabot Learning Federation we harness **oracy** and a **habits of mind** approach to develop the mathematical processes and social-emotional learning skills for mathematics. This builds on the Early Years' Characteristics of Effective Learning (CoEL) to develop all learners as proficient mathematicians ready to enjoy, engage in and think critically about mathematics.

The images on the following pages are intended to demonstrate this offer for learners; how the content of the curriculum is accessed, developed and applied through social-emotional learning skills and the mathematical processes.



Strands of mathematical proficiency

CLF Primary Mathematics curriculum offer

Being a mathematician

Social-emotional learning skills

- Mathematical resilience
- Collaboration
- Developing a sense of self and sense of place
- Thinking critically & creatively

ORACY & Mathematical Habits of Mind

Mathematical processes

- Problem solving
- Reasoning & proving
- Reflecting
- Connecting
- Communicating
- Representing
- Selecting tools & strategies

Mathematical content

National curriculum Mathematics PoS KS1 & KS2

exemplified through Ready to Progress & Curriculum Prioritisation guidance

"matters, skills and processes"

CLF EYFS Mathematics curriculum offer

Being a mathematician

Characteristics of Effective Learning

- Play & explore
- Have a go
- Active learning
- Enjoy achieving goals
- Creative & critical thinking
- Make links

ORACY
&
Mathematical Habits of Mind

Prime areas of learning

- Communication & language
- Physical development
- Personal, social & emotional development

Mathematical content

Statutory EYFS Framework

exemplified through Development Matters / Birth to 5 matters, including (but not limited to):

ELG: Number & ELG: Numerical Patterns

(*combining to develop early Number Sense*)

CLF Primary Mathematics curriculum core offer

Being a mathematician

Social-emotional learning skills

- Mathematical resilience
- Collaboration
- Developing a sense of self and sense of place
- Thinking critically & creatively

ORACY and (as a subset of our habits of mind approach) **Reasoning & Mathematical resilience**

Mathematical processes

- Problem solving
- Reasoning & proving
- Reflecting
- Connecting
- Communicating
- Representing
- Selecting tools & strategies

Core mathematical content

1. *Number sense – Cardinality (including counting & subitising), Composition & Comparison
2. *The additive relationship – the relationship between two quantities and their sum, underpinning the operations of addition and subtraction
3. *The multiplicative relationship – the relationship between two quantities and their product, underpinning the operations of multiplication and division
4. Place value – understanding that our number system uses ten digits (0-9) and a place value system that enables an infinite set of numbers to be represented as numerals.
5. Proportion – continuous quantities that cannot be represented by a whole number can be represented using decimal fractions or common fractions.
6. Lines, shapes & solids – two- and three-dimensional objects with or without curved lines and surfaces can be described, classified, and analysed by their attributes and this, rather than naming, should be the focus of teaching in early geometry.

The **core offer of the curriculum** shown here, demonstrates the key concepts (Big Ideas) that are sequenced coherently through the Ready to Progress criteria. Understanding of this mathematical content is developed as learners engage the mathematical processes and social-emotional learning skills to reason and solve problems with it.

Three core components of developing learners as mathematicians

1. Oracy

Spoken language

> *The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.*
>
> National curriculum (2014)

As the above excerpt from the Mathematics Programme of Study indicates, discussion is key to learning mathematics through reasoning and negotiating meaning. It provides opportunities for all learners to develop shared understanding of mathematical concepts and should be the primary mode for learning in every maths lesson, including paired, small group and whole class talk – led and facilitated by adults but where all learners are actively making sense of ideas for themselves.

As well as these pedagogical arguments in favour of oracy, it is also a key pillar of the CLF Disadvantage strategy; learners who are experiencing (or have previously experienced) disadvantage are likely to need additional support in developing language and the self-confidence that comes through having a voice and feeling valued. Oracy strategies that a) support vocabulary development, b) scaffold mathematical talk and c) value all learner's contributions are key to tackling some of the debilitating effects of disadvantage on both wellbeing and attainment.

To use mathematical talk for formative assessment and to support development of understanding, it is essential that all learners feel welcome to contribute ideas (in a conjecturing atmosphere), whether they match the majority of the class or not. Questions should be phrased openly where possible, teachers should avoid evaluating responses and always seek alternative views, encouraging learners to use examples and mathematical justifications to understand what is correct (i.e. not relying on the teacher's evaluation). This will enable misconceptions to be uncovered and addressed and support all learners to reason and to understand the maths for themselves. (See CLF guidance "An approach to learning through oracy" and "An approach to using oracy in maths".)

2. Mathematical Habits of Mind (2021-24 focus on reasoning)

The basis of our habits of mind approach is a set of "innate learning powers" available to all learners, through which, over time, everyone is able to make sense of their environment, understand complex concepts and learn to perform skills and processes. Examples of the power of this implicit learning system (i.e. that demonstrate the existence of these natural powers, posited by educators including Gattegno, Dewey, Polya and Mason) include the development of language and learning to walk by young children, without planned adult instruction.

Our intent is to make these innate powers explicit and harness them as efficient strategies for learning mathematics and solving problems by establishing them as habitual ways of thinking (or 'habits of mind'). They represent eight of the eleven strategies in our Habits of Mind framework:

Imagining & Expressing
Specialising & Generalising
Classifying & Characterising
Conjecturing & Convincing

In addition to these innate powers, three further strategies of ***organising*** (being systematic), ***reflecting*** and ***extending*** are intended to lead to improved problem solving and fluent understanding of content. These strategies and useful attitudes encompass the social-emotional learning skills and mathematical processes. However, as wider attitudes to mathematics – both in the UK generally and in many of our learners' specific contexts – can have a negative impact, we place added emphasis on developing the positive construct of *mathematical resilience* (see next section).

In 2023/24 the main pedagogical focus continues to be on developing teaching and learning for reasoning, which encompasses three actions (with associated HoM strategies following in brackets):

- **analysing** (classifying, characterising & organising)
- **generalising** (conjecturing & generalising)
- **justifying** (convincing)

Reasoning is essential for learners to make sense of new concepts in terms of what they already know and, as such, should feature prominently in every maths lesson (curriculum resources contain many examples of reasoning prompts and teachers can create their own using the reSolve mathematical reasoning prompts).

See Appendix A for further guidance on our habits of mind approach. Future development work will focus incrementally on remaining aspects of our habits of mind framework.

3. Mathematical resilience

Mathematical resilience acknowledges social and emotional responses to the subject that need to be anticipated and managed to support enjoyment and success. With our core pedagogies of oracy and reasoning, positive mindsets are essential for both learners and teachers; developing mathematical resilience as a positive construct is our strategy for reframing anxieties and fixed mindsets.

There are four key aspects in building mathematical resilience:

- having a growth mindset; believing that mathematical capability can improve through effort with effective strategies;
- knowing that maths can have personal value, is of value in the world and that the learner is valued as a mathematician (e.g. in their class);
- knowing that learning maths can involve struggle and 'getting stuck' and that this is to be expected rather than avoided;
- knowing how to find and enlist support to stay in the 'growth zone'

This article provides a useful model for developing mathematical resilience in the classroom.

Mathematical content

The mathematical content of the curriculum reflects the 2021 Early Years Foundation Stage and the 2014 national curriculum programmes of study (PoS).

Early Years

- Early Years teachers work within the EYFS framework, which has its own exemplification (Development Matters and/or Birth to 5 Matters) and assessment arrangements. To meet the Maths ELGs (*which do not constitute the full educational programme for maths – see next two bullet points)* Reception teachers implement Mastering Number or Number Sense maths programmes (more detail in the Factual Fluency section), which align to our curriculum and approach for factual fluency in KS1. This teacher-led learning constitutes only a small part of Reception maths learning, which takes place primarily in the wider provision.
- It is strongly suggested that Reception teachers give additional emphasis to spatial reasoning (including shape, space and measure) since there are extremely strong correlations between spatial ability (which is trainable with lasting effects) and later achievement in maths. The Early Childhood Maths Group suggests a learning trajectory for spatial reasoning and provides a toolkit to support teachers.
- Pattern is the remaining content area in EYFS maths and is the basis for all future mathematical thinking that focuses on mathematical structure and concepts, including algebraic thinking. EYFS practitioners will find useful advice and activities in this book: "The Power of Pattern".

Primary

- At primary, the **DfE/NCETM non-statutory guidance (Ready to Progress criteria, 2020)** indicates a sensible prioritisation of mathematical content, which is exemplified and sequenced through the **NCETM Curriculum Prioritisation** materials.

- Based on sound pedagogical reasons, some of the prioritised content is introduced later than in the national curriculum PoS, to create a more coherent conceptual journey (see this file for summary of moved content). From 2023/24, this will no longer cause difficulties in relation to national assessments since content is covered by the end of Year 6 (KS1 SATs and the interim Teacher Assessment Framework are no longer statutory).
- In Years 1 to 4, **it is strongly suggested that teachers adhere to the Curriculum Prioritisation overviews and teach the provided content with as much fidelity as possible** – with the caveat that Year 3 and 4 learners may have a number of gaps from EYFS or KS1, when this content was not available.
- In Years 5 and 6, most teachers will find the planning resources useful but it will be challenging to adhere to the yearly overview and teach all the provided content – most learners are likely to have too many gaps in prior learning to be able to cover the proposed content in full. Where this is the case, teachers should specifically prioritise the Ready to Progress content as the best way of preparing learners for the following year.
- Curriculum Prioritisation unit overviews may be helpful in suggesting sequence and progression, even if teachers do not use the teaching materials.

The ***suggested*** yearly overviews (long term plans) for single year group cohorts for Reception to Year 6 can be found in Appendix B – a separate powerpoint file; they provide *guidance* on sequencing and time allocation for Mastering Number (Reception) and Curriculum Prioritisation units (Y1-6).

Selecting appropriate pedagogies for types of content (arbitrary vs. conceptual)

Our core pedagogies for developing conceptual understanding and the habits of mathematicians are oracy and whole-class reasoning. However, not all mathematical content is suitable for exploring with oracy and learners should only reason about conceptual knowledge – the underlying mathematical structures which they need to make sense of. They should only be asked to use talk for learning if there is something to be discussed, negotiated, evaluated or explored e.g. concepts, conjectures and alternative solution strategies.

Some factual knowledge in the curriculum can be described as **arbitrary** – it is used by an agreed convention and there is no way of 'working it out' independently. All procedures taught are **arbitrarily chosen** (either by the teacher or by inclusion in the curriculum), since there are always multiple strategies (or multiple ways of describing them) to complete a mathematical process. Without links to conceptual understanding, arbitrary facts and arbitrarily chosen procedures need to be memorised and the act of memorising is made harder with no underlying (or associated) conceptual framework. Furthermore, without conceptual understanding, learners also need to memorise the types of situations the facts and procedures can/should be used in.

To avoid learners having to guess or invent their own:

Arbitrary factual knowledge needs to be told to learners; how to use it accurately and where it is appropriate to do so also needs to be modelled.

Arbitrarily chosen procedures need to be modelled for learners along with where it is appropriate to use them.

While these forms of direct instruction are appropriate for these kinds of content, developing understanding requires learner engagement and teacher guidance. **Oracy provides opportunities to**:

- develop understanding of the associated or underlying concepts (through reasoning)
- gather formative assessment information about what learners currently understand
- make links between arbitrary and conceptual knowledge (e.g. word roots / etymology)

Teachers should understand (and make clear to learners) what content is arbitrary and what is conceptual and how they are linked so that learners are clear which content they need to remember (or memorise) correctly and which they need to develop their understanding of through discussion of their own and others' ideas (acknowledging that understanding takes time and that misunderstandings and changes of thinking are a necessary part of the process).

All mathematical names and symbols (including numerals) are arbitrary and agreed by convention. *Number bonds and multiplication bonds are not arbitrary knowledge and should not be taught as such*, nor be simply provided only for memorisation. The most effective means of developing understanding and subsequent factual fluency is through oracy-based reasoning (derivation) based on what is already known (see next section).

The following tables aim to support teachers in understanding what content is arbitrary and what content is conceptual in the primary maths curriculum.

Identifying example number content that is arbitrary or conceptual

Arbitrary numeric symbol / name (taught best by telling)	Associated conceptual knowledge **to be developed and linked through oracy-based reasoning**	Arbitrary choice of procedure (taught best my modelling)	Underlying conceptual knowledge **to be developed and linked through oracy-based reasoning**
× is the 'multiplication symbol', written between two factors in an equation matching their product.	The multiplicative relationship between two numbers and their product.	Long multiplication algorithm (being one of many possible methods to calculate 2-digit × 2-digit multiplications)	The multiplicative relationship and the properties of commutativity, associativity and distributivity that it gives rise to.

Identifying example geometry content that is arbitrary or conceptual

Arbitrary geometric fact (taught best by telling)	Associated conceptual knowledge **to be developed and linked through oracy-based reasoning**	Arbitrary choice of geometric procedure (taught best by modelling)	Underlying conceptual knowledge **to be developed and linked through oracy-based reasoning**
The name 'square' (or e.g. the terms "equi-angular", "equilateral" and "quadrilateral")	That shapes can be classified by observable and/or measurable attributes – in this case: a closed 2-D shape with exactly four equal sides, meeting at right angles at four vertices.	Constructing a square on plain paper (can be done in a number of different ways, dependent on current understanding and skills)	Understanding of 'perpendicular' and 'parallel' as properties of lines and the concept of equivalence (for lengths and angles)

Domain	**Example arbitrary knowledge & procedures i.e. words, symbols, notation & conventions**	**Conceptual knowledge i.e. properties & relationships**
Number	• Digits/numerals & number names • linear number system increasing left to right • = < and > symbols • Naming conventions for number properties (e.g. prime, square)	• Cardinality • Composition • Comparison • Number properties
Addition & subtraction	• + and - symbols • naming conventions (e.g. subtrahend, difference) column addition & subtraction algorithms • using an empty number line to track a mental calculation (e.g. 'adding on to find a difference')	• Composition (number bonds) • Additive relationship including inverse and commutative and associative properties
Multiplication & division	• × and ÷ symbols • naming conventions (e.g. quotient, factor, product) • short/long multiplication & division algorithms • grid method multiplication • chunking for division	• Multiplicative composition (multiplication bonds) • Multiplicative relationship including inverse and commutative / associative / distributive properties
Place value	• Base ten system with left-right orientation • unit names (e.g. hundreds, tens, ones, tenths etc.) • decimal point	• Positional, additive, multiplicative aspects
Proportion (fractions)	• fraction notation including use of vinculum • decimal notation • names of fractions	• Multiplicative relationship (ratio) between numerator & denominator • Decimal equivalence / non-equivalence
Lines, shapes & solids (geometry)	• Names of shapes and units of measurement • Naming conventions (e.g. side, face, vertex etc.) • Number of degrees in a full turn • Number of units in another unit (e.g. 100cm = 1m) • Coordinate grid conventions (including e.g. x & y labels)	• 2-D & 3-D shapes can be described, classified, and analysed by their attributes

Factual Fluency

Addition & subtraction facts

Evidence demonstrates that the most effective way to learn number facts to automaticity is through reasoning/deriving strategies (I know…so I also know…) and not through rote memorisation. As this aligns with our Trust focus on developing reasoning, we have adopted two reasoning-based early number programmes (NCETM Mastering Number and Number Sense Maths) which are used for quality first teaching (i.e. only by the class teacher) in Reception, Year 1 and Year 2. Both programmes build on the core skill of subitising to develop a deep knowledge of the composition of numbers to 10, from which all future addition and subtraction calculations will be derived. Teachers requiring subject knowledge enhancement would benefit from the Early Number and Additive Reasoning Bitesize SKTM videos.

In Reception, **Mastering Number** (a free programme provided by NCETM) covers all of the number content assessed in the two Maths Early Learning Goals through provided plans for **a daily 15-20 minute teacher-led session** along with suggestions for embedding and developing number concepts through continuous and enhanced provision. The planning covers 31 weeks, teaching four days per week and therefore leaves plenty of time for teacher-led sessions on shape, space & measure (or spatial reasoning) and pattern. Number Sense Maths is similar but is a paid-for resource (with separate costs for the EY content and KS1 content).

In Year 1 and Year 2, both programmes are intended for teaching **a daily** (4 or 5 days a week), whole-class 15-minute session, **separate to the main maths lesson**. Both programmes intend to ensure that all learners will develop factual fluency in addition and subtraction facts up to 20 by Year 3. Both programmes complement the planned content in Years 1-3 of our curriculum.

In addition, Number Sense Maths is adaptable to use for early number intervention in KS2 and is strongly recommended to support the many learners with gaps following pandemic disruption to their KS1 maths learning. Mastering Number is suitable to use as a whole-cohort intervention in Year 3 (using the Year 2 materials).

Multiplication & division facts

Evidence also demonstrates that a conceptual, reasoning-based (deriving) approach to learning multiplication facts is more successful in terms of long-term retention and successful application of these facts and the Curriculum Prioritisation materials supports this through the teaching units from Year 1 (“Unitising & coin recognition”) to Year 4 (e.g. “Understanding and manipulating the multiplicative relationship”). Teachers requiring subject knowledge enhancement would benefit from the Multiplicative Reasoning Bitesize SKTM videos.

In light of the timing of the Multiplication Tables Check in Year 4, there may also need to be an element of rote recall fluency in our strategy to support learners to be successful in this assessment. All schools have access to the ‘Ashley Down Times Tables’ resources, which when implemented with high fidelity can have a positive impact on rapid recall of multiplication facts for MTC.

In 2022/23 five schools have been trialling a ‘product-based approach’ to multiplication – FunKey Maths cards – which may offer even greater benefits to factual fluency that supports multiplication & division and fractions learning in UKS2. Following a proposed follow-up study with Year 5 learners in 2023/24, we will be able to make a recommendation as to whether this approach is adopted by all schools.

Assessment

The main purpose of most assessment activities is to identify next steps for learners to develop all aspects of their mathematical proficiency. Therefore, the majority of assessment must be formative and ongoing. The different expectations in the statutory frameworks (EYFS vs. national curriculum) lead to different approaches for assessment, although the whole child perspective from Early Years is one that we are now increasingly taking account of in primary.

Early Years

When assessing children’s maths in the Early Years, it is important we take time to get to know them as learners and as mathematicians. What do they already know? What skills do they already have? How do they approach problems and what mathematical attributes do they possess? This is important in any age group, but especially with our youngest learners. Take time to observe their play; what spontaneous mathematical language are they using? Can they notice patterns? Do they have a sense of spatial awareness?

The characteristics of effective learning are fundamental to assessing young learners’ mathematics. As well as the knowledge and skills we want children to develop, building mathematical resilience, an inquisitive mind and ‘willingness to have a go’ – key habits of mind for mathematicians. Most importantly, the focus needs to be on **what learning you see embedded in their play**, rather than what they can recall in a formal situation.

Appendix C – Part 1 contains further guidance and some *indicative* milestones for maths in Nursery and Reception.

Primary

As in Early Years, understanding what children already know and can do is key. Teachers should use **pre-planning, pre-teaching assessment materials** (clearly labelled in the maths curriculum folder for each year group) to decide if learners are ready for age-related content or require pre-teaching of earlier content.

Daily teaching incorporates assessment for learning; in line with our curriculum expectations, we assess content knowledge *through application* with social-emotional learning skills and the mathematical processes. Whole-class talk, tasks and activities (whether paired, group or independent) must require thought from learners and create challenge (even struggle) to provide holistic assessment information, whilst supporting development of understanding. Daily reasoning activities will provide opportunities for developing and assessing reasoning with content and there are many examples in the materials. Teachers can create their own using the reSolve prompts / question stems (see next paragraph). For the RtP criteria that form part of our DOOYA expectations, there are suggested activities with exemplification to assist teachers in teaching and ongoing formative assessment (**“Formative assessment of RtP criteria”** which are clearly labelled in the maths curriculum folder for each year group).

Assessing whether learners are working at age-related expectations throughout the year should be based on an aggregation of ongoing formative assessments based on their confidence and success as they work through the curriculum. As well as monitoring discussion, work completed during whole-class teaching and learners' independent work, it is important to assess whether content has been understood and long-term learnt by conducting assessment after some time has passed (1-2 weeks). **Post-teach assessment materials** are clearly labelled in the maths curriculum folder for each year group and these can be used in a variety of ways (i.e. they do not have to be printed and administered as a test) to check for progress and understanding.

Accuracy and consistency of DOOYA assessments will be supported through moderation meetings (facilitated centrally and included in directed time). They are intended to support teachers making judgements as to whether pupils are on track to meet the end of year expectations outlined in the transition criteria (Appendix C – Part 2), based primarily on learners' work in class, when they demonstrate their ability to reason about age-appropriate content and to apply age-appropriate content to solve problems. This moderation work will continue to focus on formative assessment using the reSolve reasoning rubric (Appendix C – Part 3) and daily reasoning activities.

Teachers might use low-stakes summative assessments at various intervals to check for recall and accuracy of appropriate facts and skills. While these may inform aspects of teaching (and there is evidence that they may support retention), they cannot inform holistic assessment of mathematicians, since this requires the use of rich tasks that engage social-emotional learning skills and mathematical processes. Standardised assessments can only capture narrow assessment information and should only form a small part of teacher judgements, compared to formative data from day-to-day learning. Therefore, **use of timed, summative assessments should be balanced against our curriculum expectations that aims for all learners to achieve the three aims of the national curriculum and attain mathematical proficiency**. DOOYA judgements should not be based on (mainly or primarily) standardised assessments in Y1-Y5.

Expectations for learners participating in national assessments (Y4 MTC and KS2 SATs) and for teachers providing teacher assessment judgements will continue to be set by the Standards and Testing Agency and aspects of our Trust-wide work will provide support for these.

CLF Primary Mathematics Big Ideas

> "Close examination of lesson planning and teachers' thoughts about lesson planning in education systems where pupils do well reveal an intense focus on underlying knowledge structures and connections rather than the surface coherence of activities and teaching. This means that teachers are planning for what pupils will be thinking about or with, not what they will be 'doing'."
>
> OFSTED Research Review Series: Mathematics (2021)

To support teachers and learners in making connections to core mathematical structures and seeing mathematics as a connected whole, rather than a series of disparate parts, we have identified **Big Ideas** (key concepts) in mathematics, defining and describing them. Subject knowledge input and subject leader support for teachers should aim to draw attention to and exemplify these Big Ideas to enable teachers to make connections between related concepts in their teaching. Bitesize SKTM videos are available (speak to your subject leader) to support with many of the Big Ideas, highlighted according to the following key on the following pages: **Early Number highlighted in blue** **Additive reasoning highlighted in red** **Multiplicative reasoning highlighted in green**

Number: cardinality (including subitising and counting) – understanding that each numeral represents a quantity. Counting (using one-to-one correspondence) is one way of *enumerating* – establishing how many things are in a set – because the last number you say when you have counted them all correctly is the *cardinal value* of the set. However, we have an innate ability to subitise (recognise how many without counting) small quantities and this is the basis for developing early understanding of number through linking quantity, numeral and number name.

Number: composition (including conceptual subitising) – understanding that one quantity can be made up from (composed from) two or more smaller quantities. Learning to 'see' a whole number and its parts at the same time is a key development in number understanding and starts with noticing smaller parts within wholes of up to five. Deep understanding of the composition of quantities to ten is the conceptual basis for the additive relationship (see below).

Number: comparison (including comparison of quantities and position in the linear number system) – knowing which numbers are worth more or less than each other. Starting with comparing sets of objects, as learners' understanding of cardinality develops, they are able to compare abstract numbers and to understand their relative positions in a linear number system. Comparison and equivalence extend to more abstract mathematical ideas in later years.

Number: estimation – building on the innate ability to perceive differences in quantity (Approximate Number System), learners develop an increasingly accurate approximate sense of magnitude.

Number: place value – Understanding that our number system uses ten digits (0-9) and a place value system that enables an infinite set of numbers to be represented as numerals. The numeral of a 1-digit number consists of one digit; the numeral of a 2-digit number consists of two digits etc. Place value is a complex construct (with some aspects that are not developmentally appropriate content for KS1), including: positional aspect, additive aspect, base ten aspect and multiplicative aspect.

Number: the additive relationship – The additive relationship can be described as the relationship between two parts and their sum (the whole) - in other words it describes the relationships in any given composition (or decomposition) of a quantity. "*The sum of two numbers is a third number which contains as many units as the other two numbers taken together. When two of these three values are known, the third can be found*." *Ma & Kessel (2018).* If learners are encouraged to subitise (perceptually and conceptually), awareness of this relationship can be drawn out of their understanding of composition of numbers within 10.

Number: the multiplicative relationship – The multiplicative relationship describes three quantities connected by a one-to-many correspondence or ratio; when two are known the third can be found. "*The problem children face is that they view multiplication as a unary operation where you only have to really think about one number – the number being added or taken away. Seeing multiplication as repeated addition can exacerbate this unary view. Multiplication must be seen as a binary operation with two distinct inputs or elements in the process and a ratio maintained. Multiplication is framed by coordinating two ideas/quantities simultaneously.*" Ma & Kessel (2018)

Number: proportion – Continuous quantities that cannot be represented by a single whole number can be represented using decimal fractions or common fractions. Meaningful use of either format requires understanding of the quantities they represent. Fractions can also represent quantities involving a ratio between two other quantities (e.g. the concentration of orange juice in a jar can be described by the ratio of orange concentrate to water; the probability of an event can be described by the ratio between the number of favourable cases to the total number of cases)

Number: properties – for every set of numbers there are relationships that are always true and they give rise to rules that govern arithmetic and algebra. Developing age-appropriate knowledge of number properties is a key aspect of number sense.

Lines, shapes & solids – two- and three-dimensional objects with or without curved lines and surfaces can be described, classified, and analysed by their attributes and this, rather than naming, should be the focus of teaching in early geometry.

Spatial thinking – objects in space can be oriented in an infinite number of ways and an object's location in space can be described quantitatively; objects in space can be transformed in an infinite number of ways and those transformations can be described and analysed mathematically

Pattern – relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways. Seeking patterns and making conjectures based on them is the foundation of all true mathematical activity.

Data – some questions can be answered by collecting and analysing data and this data can be represented in different ways – some attributes of objects are measurable and can be quantified using unit amounts.

The history of mathematics – the discipline of mathematics has a rich history that can help us to interpret and understand current conventions and help learners to connect to their own sense of self and sense of place as mathematicians.

Appendix A: CLF Habits of Mind approach

This document (including the grids on pages two and three) identifies key attitudes, strategies, actions and questions[1] that can be developed as habits of mind in learners. It is intended to support teachers by:

- describing how learning of mathematics and learning to solve problems can be supported using habits of mind;
- defining habits of mind to model in teaching;
- suggesting observable behaviours (actions and questions) to assess problem solving and reasoning skills.

Why habits of mind?

The three aims of the national curriculum (blue shaded cells on the grids) are intended outcomes for all. Creating mathematical examples, reasoning about them (or provided examples) and evaluating their work supports learners to develop fluent conceptual understanding and problem-solving skills. We have chosen to use oracy and a habits of mind approach to support learners in these aspects of being a mathematician and develop their understanding of mathematical content.

Assessment of understanding requires the use of activities that provoke mathematical thinking and *observing the process* of enquiry not just the end result. Ongoing formative assessment is part of daily teaching and we aim to use reasoning activities and novel problems in whole-class activities to provide opportunities to assess and develop these skills along with the content of the curriculum; limiting these activities only to independent work risks many learners not achieving the aims of the curriculum and failing to develop understanding of mathematics.

Defining habits of mind as attitudes, strategies, questions and actions

We demonstrate mathematical habits of mind when we *habitually choose* **actions** and **strategies**, pose **questions** and display **attitudes** that are *productive in a mathematical context*.

Useful **attitudes**[1] are vital habits of mind to model and encourage within a classroom that has a '[conjecturing atmosphere]' that welcomes all learners' ideas. The following list is not intended to be exhaustive and may have parallels and similarities with learning powers and other metacognitive strategies already used by schools:

- **Curiosity**
- **Willingness to play** (e.g. with numbers, shapes, ideas)
- **Willingness to take risks**
- **Open to the ideas of others**
- **Perseverance, tenacity, determination**
- **Self-belief**
- **Sense of wonder**
- **Inclined to experiment/try out**

Learners' innate thinking powers (Mason et. al, 2010[2] – highlighted on the grids as four coloured pairs) are utilised unconsciously by all to learn but can be developed explicitly as problem solving **strategies** and intentionally harnessed to learn mathematics. Organising, reflecting and extending are additional strategies that can be developed over time through modelling and application to further improve understanding and problem-solving.

The **questions** can be used by teachers to encourage learners to make use of the strategies but, through modelling and opportunities to practise, they should become part of learners' own toolkits for independent problem solving. They are examples of the kinds of questions mathematicians ask themselves when presented with a problem and, as such, should be employed day to day in the learning of mathematics.

The **actions** can be modelled by adults when demonstrating a process of enquiry but can also be observed in learners as evidence of the strategies they use to solve problems.

Many thanks to Cath Gripton (University of Nottingham) for her significant contributions and support in developing these ideas.

Table 1. Developing useful habits of mind (strategies with questions)

		Strategies (including 'innate thinking powers')	Questions	
Problem solving skills	Creating	**Imagining** the ability to visualise and to think about a current unknown in the sense of "What will happen if…?"	What do I notice? What do I wonder? What might I try to find out? How can I see this? What if…? I wonder what would happen if…?	Manipulating – getting a sense of – articulating to develop an increasingly **Fluent understanding**
		Expressing communicating mathematically by any means – talking, sketches, diagrams, charts, manipulating objects	What do I know? How can I show this? How else could I show this? What am I given? What do I need to find out?	
		Specialising taking what we know (or think we know) and trying it out (testing - e.g. by creating specific examples) to gain information	What if…? I wonder what would happen if…? What am I given? What do I know? What examples can I give? Another and another? Would this be an example? What is a non-example? What changes? What stays the same?	
	Reasoning - Analysing	**Classifying** identifying specific properties of an item which define it as part of a set (e.g. 'odd' or 'square')	What's the same? What's different? Have I seen something like this before? What is this? How can I define it?	
		Characterising describing alternative properties of items that have been classified (e.g. 'odd' could also be described as 'even plus one')	What's the same? What's different? How can I show this? How else can I show it? How can I describe this? How else?	
		Organising using a system or working systematically - e.g. putting examples in order or having clear starting and finishing points to find all possibilities	What am I given? What do I know? What do I need to find out? What is a good way to start? What is the most helpful way to organise this?	
	Reasoning – generalising & justifying	**Conjecturing** *(abductive reasoning[3])* suggesting relationships and making assertions based on the evidence gathered so far	What could be going on here? Why? What will happen if I…? Is there a relationship? Will it…?	
		Convincing first oneself and then others by using mathematical arguments and justifications, including providing specific examples to prove conjectures	How do I know? How can I show this? Is it always true? Why is this true? What is really going on here?	
		Generalising identifying rules (generalisations) that can be applied to a wider range of situations, allowing us to make predictions and to use and apply knowledge to new problems	What's the same? What's different? What stays the same? What changes? What patterns can I see? Have I seen something like this before? What is this? How can I describe it in general terms? Can I show when this is true/not true?	
	Evaluating	**Reflecting[2]** *(metacognition)* evaluating ones thought processes and actions to identify next steps or future strategies	How am I approaching this? What else could I try? Have I seen something like this before? Is there a better way? Could I have done this differently? Was this the best way to start? What did I learn? How is this helpful to me?	
		Extending[2] deliberately going beyond the original problem to learn and understand more	What if…? I wonder what would happen if…? What related questions could I explore?	

Table 2. Developing useful habits of mind (strategies with actions)

		Strategies (including 'innate thinking powers')	***Example actions*** *> denote increasing sophistication but not a linear progression*	
Problem solving skills	**Creating**	**Imagining** the ability to visualise and to think about a current unknown in the sense of "What will happen if…?"	Wondering > generating possibilities > seeking pattern > (mentally) trialling imposing structure > developing a narrative > visualising > visually restructuring/reorganising	Manipulating – getting a sense of – articulating to develop an increasingly **Fluent understanding**
		Expressing communicating mathematically by any means – talking, sketches, diagrams, charts, manipulating objects	Doing > manipulating > gesturing > showing > describing > representing > recording > emphasising > demonstrating > **explaining** > **convincing > justifying > proving**	
		Specialising taking what we know (or think we know) and trying it out (testing - e.g. by creating specific examples) to gain information	Having a go > trying it out > purposeful trialling > experimenting > tinkering deliberately > changing as little as possible > **accurately defining the example space > applying own constraints**	
	Reasoning - Analysing	**Classifying** identifying specific properties of an item which define it as part of a set (e.g. 'odd' or 'square')	Noticing features/properties > comparing > same > same/different > describing what is seen > naming > comparing > noticing variance/invariance > categorising > **defining**	
		Characterising describing alternative properties of items that have been classified (e.g. 'odd' could also be described as 'even plus one')	describing what else is seen > renaming > recomparing > noticing variance/invariance > reimagining > recategorising > **redefining**	
		Organising using a system or working systematically - e.g. putting examples in order or having clear starting and finishing points to find all possibilities	Doing > ordering > mark making > deliberate recording > organising thoughts > planning (first, then) > organising information efficiently > **being systematic** taking care – checking – verifying	
	Reasoning – generalising & justifying	**Conjecturing** *(abductive reasoning[3])* suggesting relationships and making assertions based on the evidence gathered so far	Making and expressing connections > purposeful trialling (evidence of expectations) > interrogating relationships > predicting > **generating and articulating hypotheses** ("I think…" / "It's going to…")	
		Convincing first oneself and then others by using mathematical arguments and justifications, including providing specific examples to prove conjectures	Showing > describing > **explaining > convincing > justifying (***inductive reasoning[3]***) > proving (***deductive reasoning[3]***)**	
		Generalising identifying rules (generalisations) that can be applied to a wider range of situations, allowing us to make predictions and to use and apply knowledge to new problems	Repeating and forming expectations > getting a sense of > articulating (explaining in terms of what is already known) > applying (specialising) > **proving** (*deductive reasoning[3]*) defining – connecting representations – noticing variance & invariance – noticing structure – re-presenting – refining – redefining – broadening – narrowing – extending	
	Evaluating	**Reflecting[2]** *(metacognition)* evaluating ones thought processes and actions to identify next steps or future strategies	Reviewing > developing awareness > noticing > monitoring > evaluating > connecting to previous experiences parking (the first idea) – experimenting – refining – practising	
		Extending[2] deliberately going beyond the original problem to learn and understand more	Identifying similar problems – identifying related areas for exploration – constraining original task further – broadening the original task > further generalising	

Appendix C – Part 1: Early Years assessment guidance

The suggested milestones for Nursery and Reception below represent a guide to the kinds of mathematical skills and knowledge children might demonstrate on a developmental trajectory leading to achieving the Early Learning Goals for mathematics by the end of Reception. In Early Years, we value each unique child and we prioritise the characteristics of effective learning and the prime areas of learning as necessary foundations for the specific areas of learning, including mathematics.

These items below are indicative rather than definitive and should not be used as a checklist; adults should respond to the developmental needs of children rather than trying to elicit performance if they are not ready. Similarly, these ideas should not impose limits on children's ability to think critically and creatively in investigating their own ideas and solving problems they encounter. Where possible, seek evidence of children embedding these skills in their independent play in a range of situations.

Nursery	
Term 2	• recognise obvious differences in amounts (e.g. that a group of 10 is bigger than a group of 2) • show some understanding of counting behaviours, for example using number names, matching actions or clapping along with an activity • enjoy listening to and begin to join in with number and counting songs
Term 4	• use some comparative language in their play, e.g. bigger, smaller, higher, lower, more, fewer • be able to choose from a group of 2 sets when asked which has more/fewer • can complete a simple, inset puzzle, by rotating pieces to make them fit
Term 6	• begin to use 1-1 correspondence, even if the counting sequence is incorrect • subitise up to 3 • compare items by size, weight, length etc., either with gesture or language.

See next page for Reception guidance.

Reception	
On Entry	• recognise numerals with personal significance, e.g. age • spontaneously use number language in play • understands the concept of cardinality – that the last number name said tells you how many is in the set • can sort objects into matching shapes, including those with different rotations
Term 2	• consistently recognise and match numerals and quantities to 5 • instantly recognize quantities up to 5 (subitise) • use number names beyond 5, including some teens numbers • can name and talk about some simple 2d and 3d shapes
Term 4	• knows that pairs of quantities can be combined to make bigger quantities, within 5 • count objects, actions and sounds up to 10 • can describe, copy and create a two-part pattern • use language of comparison in measurements
By term 6 ELGs	Number: • Have a deep understanding of number to 10, including the composition of each number; • Subitise up to 5; • Automatically recall number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. Numerical patterns: • Verbally count beyond 20, recognising the pattern of the counting system; • Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity; • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Appendix C – Part 2: Primary DOOYA summative assessment guidance

When *creating DOOYA judgements before Term 6*, you are assessing where you think each individual **will be** in relation to the criteria **by the end of the year**.

To be assessed as '**On Track**' (O2) learners should be predicted to meet the criterion for reasoning (shaded green) and the criteria for fluency with content (shaded blue):

- In Years 1-4, learners must be predicted to be secure with both of the criteria for fluency with content;
- In both Years 5 and 6, there is one broad fluency with content criterion but in each case it must be predicted to be fully met; if necessary, assess as 'No' and detail the aspects of the criterion that will be met/not met in the 'additional notes' section.
  - e.g. for Year 5: 'Secure with 5MD-1, 5MD-2 & 5MD-3 but not likely to be secure with 5MD-4';
  - e.g. for Year 6: 'Secure with addition, subtraction & multiplication of whole numbers and decimals but not yet secure with division with remainders or where quotients have decimals'.

Securely '**On Track**' (O1) children should be predicted to **also** (*additional to the O2 criteria*) use (age-appropriate) mathematical vocabulary accurately and are likely to be more proficient reasoners (e.g. consistently achieving the 'Developing' criteria for all three actions (using 'best fit' on reSolve reasoning rubric) with some evidence of 'Consolidating' and/or 'Extending'). *This difference between O1 and O2 judgements indicates the additional support teachers may be providing their O2 learners in reasoning to be securely On Track (O1) by the end of the year.*

Learners who are '**Not Yet on Track**' must be predicted to meet the fluency with content criteria (shaded blue) with or without the vocabulary criterion. For reasoning they are likely to be assessed as mainly meeting 'Beginning' criteria on the ReSolve rubric.

Learners working '**At an Earlier Stage**' are unlikely to meet fluency or reasoning criteria.

To be assessed as '**Deepener**', as well as being predicted to meet the securely On Track criteria, learners must demonstrate a facility with solving novel problems with a range of content (beyond what has just been taught).

When *creating a DOYA judgement for end of year in Term 6*, there is only one '**On Track**' judgement and this should be assessed against the securely '**On Track**' (O1) criteria described above.

The learning dispositions criteria are not part of DOOYA judgement but will be useful information for the next teacher at transition; they may also indicate areas of focus for teaching and learning within the year with teachers considering how they can support growth mindsets and enjoyment/engagement in maths.

I am a Mathematician: Year 1 Pupil:________________ Class:_________ School:__________________

Mathematics	Key assessment criteria		Yes/No
Key knowledge	**Fluency**	Count within 100 forwards and backwards, starting from any number (**1NPV-1**)	
		Compose all numbers up to 10 from two parts and partition all numbers up to 10 into parts, including recognising odd and even numbers (**1AS-1**)	
		Uses age-appropriate mathematical vocabulary accurately	
Characteristics of effective learning	**Reasoning** (with age-appropriate content)	Reasons effectively* across a range of content	
	Problem solving (with age-appropriate content)	Has strategies to tackle and solve novel problems	
	Learning dispositions	Enjoys and engages in mathematics	
		Perseveres in the face of difficulties and evidences a growth mindset about challenges and errors	
Additional specific comments about strengths or areas for development in mathematics			

All shaded criteria must be fully met to make an 'On track' judgement at the end of the year.

*Consistently meets 'Developing' criteria on at least two of the three reasoning actions (**1** Analysing, **2** Conjecturing & generalising, **3** Convincing, justifying & proving) using the reSolve reasoning rubric.

I am a Mathematician: Year 2 Pupil:_________________ Class:_________ School:____________________

Mathematics	Key assessment criteria		Yes/No
Key knowledge	**Fluency**	Identifies or places 2-digit numbers on marked and unmarked number lines and identifies the previous and next multiples of 10. (**2NPV-2**)	
		Adds and subtracts any two 2-digit numbers within 100, applying knowledge of number bonds for all numbers up to and including ten (**2AS-4 *based on securing* 2NF-1 & 2AS-1**) ***without column methods***	
		Uses age-appropriate mathematical vocabulary accurately	
Characteristics of effective learning	**Reasoning** (with age-appropriate content)	Reasons effectively* across a range of content	
	Problem solving (with age-appropriate content)	Has strategies to tackle and solve novel problems	
	Learning dispositions	Enjoys and engages in mathematics	
		Perseveres in the face of difficulties and evidences a growth mindset about challenges and errors	
Additional specific comments about strengths or areas for development in mathematics			

All shaded criteria must be fully met to make an 'On track' judgement at the end of the year.

*Consistently meets 'Developing' criteria on at least two of the three reasoning actions (**1** Analysing, **2** Conjecturing & generalising, **3** Convincing, justifying & proving) using the reSolve reasoning rubric.

I am a Mathematician: Year 3 Pupil:__________________ Class:__________ School:_____________________

Mathematics	**Key assessment criteria**		**Yes/No**
Key knowledge	**Fluency**	Identifies or places 3-digit numbers on marked or unmarked number lines with a variety of scales, identifying previous and next multiples of 100 and 10. (**3NPV-3**)	
		Uses mental and written addition and subtraction strategies, fluently using all number bonds within 20, applying commutativity and inverse (*applying* **3AS-3** *and* **3NF-1**)	
		Uses age-appropriate mathematical vocabulary accurately	
Characteristics of effective learning	**Reasoning** (with age-appropriate content)	Reasons effectively* across a range of content	
	Problem solving (with age-appropriate content)	Has strategies to tackle and solve novel problems	
	Learning dispositions	Enjoys and engages in mathematics	
		Perseveres in the face of difficulties and evidences a growth mindset about challenges and errors	
Additional specific comments about strengths or areas for development in mathematics			

All shaded criteria must be fully met to make an 'On track' judgement at the end of the year.

*Consistently meets 'Developing' criteria on at least two of the three reasoning actions (**1** Analysing, **2** Conjecturing & generalising, **3** Convincing, justifying & proving) using the reSolve reasoning rubric.

I am a Mathematician: Year 4 Pupil:_________________ Class:_________ School:____________________

Mathematics	Key assessment criteria		Yes/No
Key knowledge	**Fluency**	Fluently adds and subtracts (*including* formal written methods) and partitions whole numbers in different ways, applying knowledge of place value (**4NPV-1, 4NPV-2**)	
		Displays automaticity in recalling all multiplication and division facts up to **10 × 10** and use them to solve multiplication and division problems within these tables (applying **4NF-1 & 4MD-2**)	
		Uses age-appropriate mathematical vocabulary accurately	
Characteristics of effective learning	**Reasoning** (with age-appropriate content)	Reasons effectively* across a range of content	
	Problem solving (with age-appropriate content)	Has strategies to tackle and solve novel problems	
	Learning dispositions	Enjoys and engages in mathematics	
		Perseveres in the face of difficulties and evidences a growth mindset about challenges and errors	
Additional specific comments about strengths or areas for development in mathematics			

All shaded criteria must be fully met to make an 'On track' judgement at the end of the year.

*Consistently meets 'Developing' criteria on at least two of the three reasoning actions (**1** Analysing, **2** Conjecturing & generalising, **3** Convincing, justifying & proving) using the reSolve reasoning rubric.

I am a Mathematician: Year 5 Pupil:_________________ Class:__________ School:____________________

Mathematics	**Key assessment criteria**		**Yes/No**
Key knowledge	**Fluency**	Fluently multiplies and divides numbers using multiplication table facts with knowledge of place value and properties of multiplication and division and applies them to solve problems, including with remainders (**5MD-1,2,3 & 4**)	
		Uses age-appropriate mathematical vocabulary accurately	
Characteristics of effective learning	**Reasoning** (with age-appropriate content)	Reasons effectively* across a range of content	
	Problem solving (with age-appropriate content)	Has strategies to tackle and solve novel problems	
	Learning dispositions	Enjoys and engages in mathematics	
		Perseveres in the face of difficulties and evidences a growth mindset about challenges and errors	
Additional specific comments about strengths or areas for development in mathematics			

Both shaded criteria must be fully met to make an 'On track' judgement at the end of the year.

*Consistently meets 'Developing' criteria on at least two of the three reasoning actions (**1** Analysing, **2** Conjecturing & generalising, **3** Convincing, justifying & proving) using the reSolve reasoning rubric.

I am a Mathematician: Year 6 Pupil:________________ Class:_________ School:___________________

Mathematics	Key assessment criteria		Yes/No
Key knowledge	**Fluency**	Calculates fluently with whole numbers (using all four operations) and decimals (including **6AS/MD-1 & 6AS/MD-2**)	
		Uses age-appropriate mathematical vocabulary accurately	
Characteristics of effective learning	**Reasoning** (with age-appropriate content)	Reasons effectively* across a range of content	
	Problem solving (with age-appropriate content)	Has strategies to tackle and solve novel problems	
	Learning dispositions	Enjoys and engages in mathematics	
		Perseveres in the face of difficulties and evidences a growth mindset about challenges and errors	
Additional specific comments about strengths or areas for development in mathematics			

Both shaded criteria must be fully met to make an 'On track' judgement at the end of the year.

*Consistently meets 'Developing' criteria on at least two of the three reasoning actions (**1** Analysing, **2** Conjecturing & generalising, **3** Convincing, justifying & proving) using the reSolve reasoning rubric.

Appendix C – Part 3 CLF Reasoning Rubric

(based on the reSolve reasoning rubric)

	ANALYSING	CONJECTURING & GENERALISING	CONVINCING, JUSTIFYING & PROVING
NOT EVIDENT	• Does not notice numerical or spatial structure of examples or cases. • Attends to non-mathematical aspects of the examples or cases.	• Does not communicate a common property or rule (conjecture) for a pattern.	• Does not convince or justify. • Appeals to teacher or others.

BEGINNING	• Notices similarities across examples
	• Recalls random known facts related to the examples
	• Recalls and repeats patterns displayed visually or through use of materials.
	• Attempts to sort cases based on a common property
		• Draws attention to or attempts to communicate a common property or repeated components of a pattern using:
		  ○ body language (gesture),
		  ○ drawing,
		  ○ concrete materials,
		  ○ counting *or*
		  ○ oral language (metaphors)
			• Describes what they did and why it may or may not be correct
			• Recognises what is correct or incorrect using materials, objects or words.
			• Makes judgements based on simple criteria such as known facts
			• The argument is not coherent or does not include all steps in the reasoning process

DEVELOPING	• Notices a common numerical or spatial property
	• Recalls and repeats patterns using numerical structure or spatial structure.
	• Sorts and classifies cases according to a common property
	• Orders cases to show what is the same or stays the same and what is different or changes.
	• Describes the case or pattern by labelling the category or sequence.
		• Communicates a rule (conjecture) about a:
		  ○ *property* using words, diagrams or equations
		  ○ *pattern* using words, diagrams to show recursion or equations to communicate the pattern as repeated addition
		• Records other cases that fit the rule (conjecture) or extends the pattern using the rule.
			• Attempts to verify by testing cases or explaining the meaning of a conjecture using one example
			• Detects and corrects errors and inconsistencies using materials, diagrams and informal written methods.
			• Starting statements in a logical argument are correct and accepted by the classroom.

CONSOLIDATING	• Notices more than one common property by systematically generating further cases and/or listing and considering a range of known facts or properties.
	• Repeats and extends patterns using both the numerical and spatial structure.
	• Searches for and produces examples:
	  ○ using tools, technology and modelling
	• Makes predictions about other cases:
	  ○ with the same property
	  ○ included in the pattern
		• Generalises: communicates a rule (conjecture) using mathematical terms, symbols or diagrams (e.g. an equation or labelled geometric diagram)
		• Explains what the rule (conjecture) means using one example
		• Extends the pattern using an example to explain how the rule works
			• Verifies truth of statements by using a common property, rule or known facts that confirm each case. May use materials and informal methods.
			• Refutes a claim by using a counter example.
			• Uses a correct logical argument that has a complete chain of reasoning and uses words such as 'because', 'if…then…', 'therefore', 'and so', 'that leads to'
			• Extends the generalisation using a logical argument

EXTENDING	• Notices and explores relationships between:
	  ○ common properties
	  ○ numerical structures of pattern
		• Generalises: communicates the rule (conjecture) using mathematical symbols
		• Applies the rule to find further examples or cases.
		• Generalises properties by forming a statement about the relationship between common properties.
		• Compares different symbolic expressions used to define the same pattern to show equivalence.
			• Uses a watertight logical argument that is mathematically sound and leaves nothing unexplained
			• Verifies that the statement is true or the generalisation holds for *all* cases using logical argument

The use of this rubric will be supported through the Trust-wide moderation sessions. There is an A3 version for easier use.